Development of Anisotropic Hyperelastic Model Considering Stress Softening

Akihiro Matsuda
University of Tsukuba
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Introduction

- Fiber-Reinforced Rubber (FRR) have,
  - Anisotropic mechanical characteristics
  - Stiffness softening according to maximum stretch (Mullins effect)
  - Large deformation gives damage to the internal structures

Mullins effect

FRR
Objective

- Develop anisotropic hyperelastic model considering stress softening (Mullins effect)
  - Evaluate applicability of proposed model to FE analysis
Hyperelastic Modeling

- Anisotropic hyperelastic model
  - The matrix about the stress $S$ is obtained from strain energy function

\[
S = 2 \frac{\partial W}{\partial C}
\]

\[
W = W_{iso}(C) + W_{ani}(C, M)
\]

- Mooney-Rivlin Model was applied to isotropic part

\[
W_{iso} = c_1(\tilde{I}_1 - 3) + c_2(\tilde{I}_2 - 3)
\]
Anisotropic Part

- Introduce information of directions of reinforced fibers

\[ J_4^{(a)} = F(n^{(a)} \otimes n^{(a)})F = C : M^{(a)} \]
\[ J_5^{(a)} = C^2 : M^{(a)} \]

- Here, a=1 means warp fiber, a=2 means weft fiber, respectively
- \( n^{(a)} \) is unit vector of which direction is same as reinforced fibers
- \( J_4 \) means square of fiber stretch
- Angle between \( n^{(1)} \) and \( n^{(2)} \) is 90° (Orthotropic)

- Another invariants are introduced

\[ K_1^{(a)} = J_5^{(a)} - I_1 J_4^{(a)} + I_2 = \text{cof}(C) : M^{(a)} \]

- \( J_4 \) and \( K_1 \) are applied to stored energy
Anisotropic Part

Use following model for anisotropic component

\[ W_{ani} = \frac{1}{4} \sum_{a=1}^{2} \left[ \frac{C_J^{(a)}}{d_J^{(a)}} \left\{ \left( J_4^{(a)} \right)^{d_J^{(a)}} - 1 \right\} + \frac{C_K^{(a)}}{d_K^{(a)}} \left\{ \left( K_1^{(a)} \right)^{d_K^{(a)}} - 1 \right\} \right] \]

where

- \( d_J^{(a)} \) and \( d_K^{(a)} \) are model parameters
- \( J_4^{(a)} \) and \( K_1^{(a)} \) are related to the deformation of the yarn

\[ - \frac{1}{4} \sum_{a=1}^{2} \left( C_J^{(a)} - C_K^{(a)} \right) \left( J_4^{(a)} - 1 \right) - C_K^{(a)} \ln (J) \]

\[ + \frac{1}{4} \sum_{a=1}^{2} C_{Jn}^{(a)} \left\{ J_4^{(a)} - 1 + \frac{\exp \left\{ d_{Jn}^{(a)} \left( 1 - J_4^{(a)} \right) \right\} - 1}{d_{Jn}^{(a)}} \right\} \]

The stress in the small stretch

\[ \sum_{a=1}^{2} \left( W_{warp}^{(a)} + W_{weft}^{(a)} \right) = W_{ani}^{(a)} \]

Material Constants

- \( J_4^{(a)} (C, M^{(a)}) \) : Tensile deformation
- \( K_1^{(a)} (C, M^{(a)}) \) : Shear deformation
Modeling of Stress-Softening

Anisotropic part was modified by Stress-Softening function

\[ W'_{ani} = S^{(1)}_f W_{warp} + S^{(2)}_f W_{weft} \]

\[ S^{(a)}_f = 1 - \alpha_a \left[ 1 - \exp \left\{ - \gamma_a \left( J_{4_{\text{max}}}^{(a)} - 1 \right) \right\} \right] \quad (a = 1, 2) \]
Biaxial loading test for rubber matrix

\[ W_{iso} = c_1(\bar{I}_1 - 3) + c_2(\bar{I}_2 - 3) \]

<table>
<thead>
<tr>
<th></th>
<th>c₁</th>
<th>c₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrile rubber</td>
<td>1.29</td>
<td>0.148</td>
</tr>
</tbody>
</table>

The stress of tension side and that of fixed side
Uniaxial cyclic tensile loading test

- **Specimen**
  - Nitrile rubber reinforced by cotton fiber (use same nitrile rubber to biaxial loading test)
  - 20mm × 40mm × 2mm
  - Fiber orientation angle θ = 0, 45, 90

- **Test condition**
  - Velocity was 0.1% / sec
  - 5 cycles of tensile deformation were given in each maximum stretch
Uniaxial cyclic tensile loading test

Nominal Stress (MPa)

Nominal Stretch

\( \theta = 0 \)

\( \theta = 90 \)

\( \theta = 45 \)

Mullins effect
Identification of material constants

<table>
<thead>
<tr>
<th>$a$</th>
<th>$c_J^{(a)}$</th>
<th>$d_J^{(a)}$</th>
<th>$c_K^{(a)}$</th>
<th>$d_K^{(a)}$</th>
<th>$c_{Jn}^{(a)}$</th>
<th>$d_{Jn}^{(a)}$</th>
<th>$\alpha^{(a)}$</th>
<th>$\gamma^{(a)}$</th>
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<tbody>
<tr>
<td>1</td>
<td>29.0</td>
<td>36.5</td>
<td>1.0</td>
<td>1.0</td>
<td>25.0</td>
<td>30.0</td>
<td>1.0</td>
<td>23.0</td>
</tr>
<tr>
<td>2</td>
<td>1.89</td>
<td>24.1</td>
<td>1.0</td>
<td>1.0</td>
<td>1.55</td>
<td>319</td>
<td>1.0</td>
<td>14.3</td>
</tr>
</tbody>
</table>

Relationships between stress and strain ($\theta = 0$)

Relationships between stress and strain ($\theta = 90$)
Finite Element Method

- **Size and mesh**
  - 20mm × 40mm × 2mm
  - 3-dimensional 8 node element

- **Boundary condition**
  - Top and bottom surface were constrained
  - Tensional displacement were applied
Finite Element Method

Experienced the stretch of 1.40

θ = 0

1.30

θ = 45

1.30

θ = 90

1.20

1.15

1.20

1.25
Results of FE simulation

- Results of FEM simulations are compared with tensile loading test results
- The proposed model is able to represent anisotropic and stress softening of fiber reinforced rubber

Comparison of the experimental data and FEM results data
Stability of FE simulation

- FEM code was developed
  - 3-dimensional solid element
  - Displacement/pressure mixed method
  - Newton-Raphson method for iteration

- Boundary condition
  - FEM model is 20mmx20mmx20mm
  - Upper and bottom surface were constrained
  - Cyclic 40% of shear deformation was applied
Shear Deformation

\[ \alpha = 90^\circ \]
- 20% (1st) 40% (1st) 20% (2nd) 40% (2nd)

\[ \alpha = 45^\circ \]
- 20% (1st) 40% (1st) 20% (2nd) 40% (2nd)

Warp

[Color scale from 0.00MPa to 400.00MPa]
Conclusion

- A stress-softening model for anisotropic hyperelasticity was shown in this study.

- Anisotropic mechanical characteristics and stress softening (Mullins effect) of FRR were able to be represented by proposed model.

- Proposed model is possible to apply to FEM analysis and FEM simulation shows high robustness.
Thank you for your attention
Stress of Hyperelasticity

- Stress of isotropic hyperelasticity is given by

\[
S = 2 \frac{\partial \psi(\bar{I}_1, \bar{I}_2, I_3)}{\partial C}
\]

- **S**: 2nd Piola Kichhoff stress
- **C**: Right-Green deformation tensor
- **\( \psi \)**: Stored energy function
- **\( I_1, I_2, I_3 \)**: 1st, 2nd and 3rd prime invariant of \( C \) tensor
- **\( \bar{I}_1 = I_3^{-1/3} I_1 \)**: Modified 1st invariant
- **\( \bar{I}_2 = I_3^{-2/3} I_2 \)**: Modified 2nd invariant
Fiber Reinforced Rubber (FRR)

- Fiber reinforced rubber (FRR) is a composite of rubber matrix and reinforcing fibers.
- FRR shows high strength and high flexibility.
- Anisotropic mechanical characteristics in large deformation.
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Tensile Tests

- Bi-directional tensile loading test
  - To evaluate the damage independence of warp yarn and weft yarn

- Uniaxial cyclic tensile loading test
  - To evaluate the damage effect at each stretch
  - To do the identification of material constants
Bi-directional tensile loading test

- Nitrile rubber reinforced by cotton fiber
  - $\theta = 0, 45, 90$ (\(\theta\) means fiber orientation angle)

1. Loading up to a certain stress value in orthogonal direction
2. Providing the relaxation time for an hour
3. Cutting the specimen in rectangular shape and loading
Bi-directional tensile loading test

For example, the specimen of $\theta = 0$

Cut

Weft yarn is stretched

Warp

Weft yarn

stretched
Bi-directional tensile loading test

Nominal Stress : \( \theta = 0 \)

Nominal Stress : \( \theta = 45 \)

Load = \( \frac{\text{Initial cross-section area}}{\text{Initial length}} \)

Nominal Stress :

Nominal Stretch :

Loading up to 0 MPa in orthogonal direction
Loading up to 10 MPa in orthogonal direction
Loading up to 20 MPa in orthogonal direction
Loading up to 30 MPa in orthogonal direction
Bi-directional tensile loading test

Consideration

- The relationship between the warp direction and the weft direction.
- The structure of weave yarn in the specimen

Warp yarn and weft yarn shows damage independence.
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Stress of Hyperelasticity

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- **\( \bar{I}_1 = I_3^{-1/3} I_1 \)**: Modified 1st invariant
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Invariants for Anisotropic Hyperelasticity

- Introduce information of directions of reinforced fibers

\[ J_4^{(a)} = F \left( n^{(a)} \otimes n^{(a)} \right) F = C : M^{(a)} \]
\[ J_5^{(a)} = C^2 : M^{(a)} \]
  - Here, a=1,2, respectively
  - \( n^{(a)} \) is unit vector of which direction is same as reinforced fibers
  - \( J_4 \) means square of fiber stretch
  - Angle between \( n^{(1)} \) and \( n^{(2)} \) is 90° (Orthotropic)

- Another invariants are introduced

\[ K_1^{(a)} = J_5^{(a)} - I_1 J_4^{(a)} + I_2 = \text{cof} \left( C \right) : M^{(a)} \]

- \( J_4 \) and \( K_1 \) are applied to stored energy
Stored Energy of Anisotropic Hyperelasticity

- Stored energy function of anisotropic hyperelasticity

\[ \psi = \psi_{iso}(\bar{I}_1, \bar{I}_2) + \psi_{ani}(J_4^{(a)}, K_1^{(a)}) + \psi_{vol}(I_3) \]

- Isotropic part (Rubber matrix)

\[ \psi_{iso}(\bar{I}_1, \bar{I}_2) = C_1(\bar{I}_1 - 3) + C_2(\bar{I}_2 - 3) \]

- The Mooney-Rivline model was applied to rubber matrix
Material Test of Rubber Matrix

- Biaxial loading test for Nitric rubber matrix
- Rubber sheet specimen(110mmx110mmx3mm)
  - One side was loaded to 200% stretch.
  - Other side was kept initial length
  - Two stress $\sigma_1$ and $\sigma_2$ were approximated by Mooney Rivline model ($C_1=1.28\text{MPa}$, $C_2=0.15\text{MPa}$)
Tensile Loading Test of FRR

- Tensile loading tests of fiber reinforced rubber were conducted
  - Tensile loading test specimen (120mm x 20mm x 3mm)
  - Aluminum plates (20mm x 20mm x 2mm) were bonded at fixing part
  - $\alpha$ is angle between warp and tensile direction

- Tensile tests was conducted under the condition of $\alpha=0, 15, 30, 45, 60, 75, 90$
Stiffness in the direction of warp and weft show different
Stress-strain curve of $\alpha=30, 45, 60$ shows softening between 1.05~1.2 of stretch
Stored Energy of Anisotropic Part

Anisotropic part was modified to consider different strength of warp and weft

\[ \psi_{ani}(J_4^{(a)}, K_1^{(a)}) = \psi_{add} + \psi_{Asai} + \psi_{Its} \]

We apply additional function to the Asai and the Itskov models for better approximation in small deformation

\[ \psi_{add} = \frac{1}{4} \sum_{a=1}^{2} C_{Ja}^{(a)} \left[ J_4^{(a)} - 1 + \exp \left\{ \frac{d_{Ja}^{(a)} (1 - J_4^{(a)})}{d_{Ja}^{(a)}} \right\} - 1 \right] \]

Behavior in small stretch region is improved by the proposed model
Propose model is applicable to the reinforced rubber for seals in generator
Stability of FEM Code

- FEM code was developed
  - 3-dimensional solid element
  - Displacement/pressure mixed method
  - Newton-Raphson method for iteration

- Boundary condition
  - FEM model is 20mmx20mmx20mm
  - Upper and bottom surface were constrained
Tensile Deformation

- Mean stress distributions of tensile deformation of 1.5 stretch

- Mean stress distribution of cross section

θ=0°  θ=15°  θ=30°  θ=45°
Shear Deformation

- Initial
- $\alpha = 0^\circ$
- $\alpha = 15^\circ$
- $\alpha = 30^\circ$
- $\alpha = 45^\circ$
- $\alpha = 60^\circ$
- $\alpha = 75^\circ$
- $\alpha = 90^\circ$

Warp

Legend:
- 0.00MPa
- 400.00MPa
THANK YOU FOR YOUR ATTENTION

- Asai model

\[ \psi_{\text{asai}} = \frac{1}{4} \sum_{a=1}^{2} \left[ \frac{C^{(a)}_J}{d^{(a)}_J} \left( J^{(a)}_4 \right)^{d^{(a)}_J} - 1 \right] + \frac{C^{(a)}_K}{d^{(a)}_K} \left( K^{(a)}_1 \right)^{d^{(a)}_K} - 1 \]

\[ - \left\{ \left( C^{(a)}_J - C^{(a)}_K \right) \left( J^{(a)}_4 - 1 \right) + C^{(a)}_K \ln \left( J^2 \right) \right\} \]

- Itskov model

\[ \psi_{\text{Its}} = \frac{1}{4} \sum_{a=1}^{2} \left[ C^{(a)}_I \left( J^{(a)}_4 - 1 \right) + \left( K^{(a)}_1 - 1 \right) - \ln \left( J^2 \right) \right] \]